# Day-ahead stochastic scheduling model considering the reserve of large-scale HPs

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Nomenclature

**Abbreviations**

CHPCombined heat and power

EPSElectrical power system

DHSDistrict heat system

IEHSIntegrated electricity and heat system

HPHeat pump

STStorage tank

WPWater pump

**Indices and sets**

Set of indicesof time periods

Set of indicesof thermal power units

Set of indicesof CHP units

Set of indicesof water pumps

Set of indicesof wind farms

Set of indicesof HPs

Set of indicesof STs

Set of indicesof buses in the EPS

Set of indicesof transmission lines in the EPS

Set of indicesof wind power scenarios

Set of indicesof pipes in the DHS

Set of indicesof nodes in the DHS

Set of indicesof nodes with heat sources

Set of indicesof nodes with heat exchange stations

Set of indicesof nodes with heat loads

Set of indicesof thermal units/CHP units/wind farms/HP located on bus *d*

Set of indicesof segments of linearized fuel cost function of thermal units

Set of indices of start nodes and end nodes of pipe *b*

Set of indicesof pipes to/from node *n*

**Parameters**

 Start-up cost of thermal/CHP unit *i*

Penalty price of wind power curtailment

Penalty price of load shedding

Capacity price of upward/downward following reserve of thermal unit *i*

Price of upward/downward following reserve deployment for thermal unit *i*

Price of upward/downward following reserve deployment for CHP unit *i*

Capacity price of upward/downward following reserve of CHP unit *i*

 Capacity price of upward/downward following reserve of large-scale HP *i*

 Specific heat capacity of water

 Coefficient of performance of HP

Fuel cost corresponding to the minimum output level of thermal unit *i*

** Fuel cost of vertices A/B/C/D for CHP unit *i*

 Capacity of transmission line *l*

Distribution factor of the thermal power unit/CHP unit/wind farm/HP/load at bus *d* on line *l*

** Maximum/minimum energy storage of STs

** Maximum charging and discharging rate of STs

** Heat production of vertices A/B/C/D for CHP unit *i*

 Slope of segment *k* of linearized fuel cost function of thermal unit *i*

 Load at bus *i* in period *t*

 Length of pipeline *b*

 Mass flow rate in heat sources/heat exchange stations

 Mass flow rate of pipeline *b* inperiod *t* in supply/return network

Maximum/minimum output of thermal unit *i*

Maximum/minimum output of CHP unit *i*

Maximum value of segment *k* of thermal unit *i*

** Power production of vertices A/B/C/D for CHP unit *i*

** Maximum power consumption of HP *i*

** Maximum/minimum power consumption of WP *i*

 Downward regulating reserve capacity of thermal power unit *i* in period *t*

 Upward regulating reserve capacity of thermal power unit *i* in period *t*

 Downward regulating reserve capacity of large-scale HP *i* in period *t*

 Upward regulating reserve capacity of large-scale HP *i* in period *t*

 Maximum/minimum temperature of supply/return pipes

 Ambient temperature in period *t*

 Minimum start-up time of thermal/CHP unit *i*

 Minimum shut-down time of thermal/CHP unit *i*

 Upward/downward ramping rate of thermal/CHP unit *i*

 Forecast value of wind farm *i* in period *t*

Wind power output for wind farm *i* in scenario *s* in period *t*

 Heat transfer coefficient of pipes

 Efficiency of water pump *i*

 Density of water

**Variables**

** Total operational cost

** The first-stage operational cost

** The second-stage operational costunder scenario *s*

 The first-stage following reserve capacity cost of all large-scale HPs.

 The second-stage following reserve deployment cost of all large-scale HPs under scenario *s*. The total operational cost of all thermal/CHP units in the first stage

 The regulation cost of all thermal/CHP units under scenario *s* in the second stage

 Penalty cost of wind power curtailment for all wind farms in the first stage

 Penalty cost of wind power curtailment for all wind farms under scenario *s* in the second stage

 Penalty cost of load shedding under scenario *s* in the second stage

** Operational cost of CHP unit *i* in period *t*

 Heat energy level of ST *i* in period *t*

 Heat energy level of ST *i* in period *t* under scenario *s*

 Heat production of CHP unit *i* in period *t*

 Heat production of HP *i* in period *t*

 Power production of thermal/CHP unit *i* in period *t*

 The scheduled value of segment *k* for thermal unit *i* in period *t*

 Electricity consumption of water pump *i* in period *t*

 Electricity consumption of HP *i* in period *t*

 Pressure of node *n* in period *t* for supply/return network

 Inlet temperature of pipe *b* in period *t*

 Outlet temperature of pipe *b* in period *t*

 Temperature of node *n* in period *t*

 Inlet temperature of pipe *b* in period *t* under scenario *s*

 Outlet temperature of pipe *b* in period *t* under scenario *s*

 Temperature of node *n* in period *t* under scenario *s*

 Upward/downward following reserve capacity of large-scale HP *i* in period *t*

 Upward/downward following reserve capacity of thermal unit *i* in period *t*

 Upward/downward following reserve capacity of CHP unit *i* in period *t*

 Start-up indicator of thermal/CHP unit *i* in period *t*

** Probability value used to sample relative to random variable **

 Shut off indicator of thermal/CHP unit *i* in period *t*

** Random variable relative to the actual output of wind farm *j* in period *t* under scenario *s*

 Random variable relative to the forecast output of wind farm *j* in period *t*

 On-off status of thermal/CHP unit *i* in period *t*

 On-off status of HPs

 Coefficient of vertice *A/B/C/D* of CHP unit *i* in period *t*

 Coefficient regulation of vertice *j* for CHP unit *i* in period *t* under scenario *s* related to upward/downward following reserve

 Charging/discharging heat energy of ST *i* in period *t*

 Charging/discharging heat energy of ST *i* in period *t* under scenario *s*

 Heat regulation of large-scale HP *i* related to upward/downward regulating reserve deployment in period *t* under scenario *s*

 Heat regulation of large-scale HP *i* related to upward/downward following reserve deployment in period *t* under scenario *s*

 Regulation of heat production for CHP unit *i* in period *t* under scenario *s* relative to

 Load shedding of bus *i* in period *t* under scenario *s*

 Regulation of electricity consumption of HP *i* in period *t* under scenario *s*

 Upward/downward following reserve deployment for HP *i* in period *t* under scenario *s*

 Upward/downward regulating reserve deployment for HP *i* in period *t* under scenario *s*

 Upward/downward following reserve deployment for thermal unit *i* in period *t* under scenario *s*

 Upward/downward regulating reserve deployment for thermal unit *i* in period *t* under scenario *s*

 Upward/downward following reserve deployment for CHP unit *i* in period *t* under scenario *s*

 Wind power curtailment in the base case for wind farm *i* in period *t*

 Wind power curtailment for wind farm *i* in period *t* under scenario *s*

1. Mathematical formulation of day-ahead stochastic optimal operation of IEHS

*1.1 Objective function of IEHS*

The objective function of the proposed dispatching scheme can be expressed as (1). It minimizes the total operational cost in two stages, i.e., the day-ahead operational cost and real-time expected cost.

 (1)

 (2)

*1.2 Operational constraints of IEHS*

The optimal operation problem of the IEHS includes the day-ahead operational constraints and real-time regulation constraints. They are described in the following subsections.

1.2.1 Day-ahead operational constraints of EPS

* Power balance constraint

 (3)

* Constraints of thermal units and CHP units

 (4)

 (5)

 (6)

 (7)

 (8)  (9)

 (10)

 (11)

 (12)

 (13)

 (14)

 (15)

 (16)

 (17)

 (18)

 (19)

 (20)

 (21)

where (4) represent the output limits of thermal units, which considers the regulating reserve capacity; (5)-(6) represent the piece-wise linearization of each thermal unit; (7)-(8) describe the upward and downward ramping rate limits of the thermal units; (9)-(10) limit the following reserve capacity of thermal units; (11) represent the power production of CHP units based on vertex coefficients of convex-hull feasible region; (12) combines the sum of vertex coefficients with the on/off status of CHP units; (13) limits the range of coefficients of the vertices of the feasible region for CHP units; (14)-(15) describe the upward and downward ramping rate limits of the CHP units; (16)-(17) limit the following reserve capacity of CHP units; (18)-(21) model the changes of the on/off status of the thermal units and CHP units and their minimum up and downtime requirements.

* Constraints of HPs/WPs

 (22)

 (23)

 (24)

 (25)

where (22) represents the power consumption constraint of HPs with the on/off status, which considers the coupling with upward and downward regulating reserve capacities; (23) and (24) limit the downward and upward following reserve capacities, respectively; (25) represents the power consumption limit of water pumps (WPs) to sustain a certain level of pressure for the water cycle.

* Constraints of wind power curtailment

 (26)

* Constraints of transmission lines

(27)

1.2.2 Day-ahead operational constraints of DHS

* Heat output and heat exchange

 (28)

 (29)

where (28) reflects the heat balance of the heat network; and (29) describes that the heat exchange in the heat station is equal to the heat demand.

* Constraints of thermal process

 (30)

 (31)

 (32)

where (30) limits the temperature range of supply and return pipes; (31) represents the temperature drop along the pipe; and (32) presents the water mixture process.

* Constraints of hydraulic process

 (33)

 (34)

 (35)

where (33) represents that the pressure between supply and return water in a heat exchange station is required to be larger than a specified level to sustain the mass flow; (34) describes that the power consumption of WPs is proportional to the pressure difference; (35) represents that the pressure difference between the two ends of a pipe is proportional to the square of mass flow rate.

* Heat production constraints of CHP/HPs

 (36)

 (37)

where (36) represents the heat production of CHP units; and (37) describes the heat production of large-scale HPs is coupled with their power consumptions.

* Constraints of STs

 (38)

 (39)

 (40)

where (38) limits the maximum charging/discharging rate; (39) reflects the changes in energy level; and (40) constrains the capacity of heat storage.

1.2.3 Real-time operational constraints of EPS

In this section, the reserve from the thermal units, CHP units, and large-scale HPs, is deployed to balance the wind power deviations in scenarios. As the deployed reserve from those devices is tightly coupled with the base plan in the first stage, the linkage constraints between the two stages are modeled for each device.

* Power re-balance constraints

 (41)

* The linkage constraints of CHP units between two-stages

 (42)

 (43)

 (44)

 (45)

 (46)

 (47)

 (48)

 (49)

 (50)

where (42)-(50) represent the reserve deployment of CHP units in each scenario. Among (42)-(50), (42)-(43) ensure the upward and downward following reserve deployments in each scenario no more than the following reserve capacity; (44)-(45) utilize the linear combination of convex-hull vertices to represent the reserve deployment in each scenario; (46)-(47) represent the relationship of the regulation coefficient of four vertices, which can be deduced from ; (48)-(50) limit the range of regulation coefficients.

* The linkage constraints of thermal units/HPs between two stages

 (51)

 (52)

 (53)

 (54)

where (51) and (52) represent the deployed following reserve and regulating reserve of thermal units in each scenario are no more than the following reserve capacity and regulating reserve capacity in the first stage, respectively; (53) and (54) describe that the following reserve and regulating reserve deployments of large-scale HPs are no more than the scheduled following and regulating reserve capacities in the first stage, respectively.

* Wind power curtailment and load shedding under each scenario





* Constraints of transmission lines under each scenario



* Ramping constraints after reserve deployment under each scenario





 (60)

 (61)

1.2.4 Real-time operational constraints of DHS

* Heat re-balance constraints under each scenario

 (62)

 (63)

* Thermal process under each scenario

 (64)

 (65)

 (66)

where (64)-(66) are similar to (30)-(32), which represent the thermal process in each scenario.

* Heat regulation of CHPs/HPs

 (67)

 (68)

 (69)

 (70)

 (71)

 (72)

where (67)-(68) describe that the heat regulation of CHP units are related to their upward and downward following reserve deployments in each scenario; (69)-(72) represent that the heat regulation of large-scale HPs are related to their upward/downward following and regulating reserve deployment.

* Heat regulation of STs under each scenario

 (73)

 (74)

 (75)

where (73)-(75) are similar to (38)-(40), representing the charging/discharging rate, changes of heat energy level, and heat storage capacity under each scenario.